

## 15 Perplexing Problems

Warmups:

a) Suppose you are an astronaut who is on a space walk. A 100-foot tether is keeping you attached to the space shuttle and the only thing you are carrying is a battery-operated fan. Suddenly, a meteor slices through your tether and you are left stranded 100 feet from the safety of the shuttle. How would you get back to the shuttle?

b) A ball and a glove together cost \$10. If the glove cost \$9 more than the ball, what is the cost of the ball?

c) Some scientists estimate that the universe contains  $10^{87}$  atoms. How many atoms are there in one tenth of the universe?

1) An eccentric mathematician gives one die to a student and declares that if the student rolls a “4”, he will give the student \$1,000,000. To make the student more likely to win the \$1,000,000, the mathematician decides that the student will get three rolls of the die. If any roll is a “4”, the money is hers. What is the probability that the student will win \$1,000,000?

2) A well known game show had contestants try to win valuable prizes by picking one of three doors. The game show host knew where the valuable prize was, so after a contestant made her pick, the host would open a door he knew didn't contain the prize and show the contestant what was behind that door. He would then ask the contestant if she wanted to trade her door for the remaining door. What are the chances that the contestant will win if she keeps her original door? What are the chances that she will win if she switches doors?

3) Which causes more pressure on a rope?

a) Ten 200-pound people on one side of a rope who are pulling with all their might with the other side attached to a tree.

b) Ten 200-pound people on one side of a rope with another ten 200-pound people on the other side. Both sides are pulling with all their might.

3) Which collision would be more violent?

a) A car traveling 60 miles per hour hitting a brick wall.

b) Two identical cars traveling in opposite directions at 60 miles per hour and hitting each other head on.

4) You meet a long lost friend and find out that she has two children. You ask her the following question: “Is at least one of your children a boy?”

Your friend responds that yes, at least one of her children is a boy. What are the chances that her other child is also a boy?

5) Phil hiked up a mountain at 3 miles per hour and then went down the same trail at a speed of 6 miles per hour. What was Phil's average speed for his hike?

6) Three coins are in a hat. One of the coins has heads on both sides, while another coin has tails on both sides. The third coin has heads on one side and tails on the other. You pick a coin and look at one side and see that it is heads. What is the probability that the other side will be heads?

7) Joseph can paint  $n$  cars in  $t$  hours. How long does it take for Joseph to paint one car?

8) It takes Sara 2 hours to paint a fence and Daniel 4 hours to paint the same fence. How long will it take them to paint the fence if they both work together?

9) Approve the following math for this month's publication of AARP.

**It is known that 45% of men snore and 25% of women snore. A doctor looked at these numbers and made the following statement:**

**“If you put a man and a woman together, there is a 70% chance that someone is snoring.”**

10) Mark started a new job at a salary of \$10,000 per year. He was given a choice as to how he would receive raises in salary during his time working with the company. What option should Mark choose to receive the most amount of money if he plans to work 10 years? Choice 1: A raise of \$1000 each year

Choice 2: A raise of \$300 each six months

11) You see two boxes on a table. You are told that one box has twice as much money as the other box. You open the box of your choice and find that it contains \$900. Now you are given the opportunity to switch boxes. Do the laws of probability suggest that you switch?

12) Two basketball players are each 9 for 30 in shooting baskets. Player A then goes 1 for 10 on his next 10 shots. Player B does much better and sinks 6 of his next 30 shots. Even though Player B did much better, both players ended up shooting 25%.

Player A: 10 for 40

Player B: 15 for 60

13) If the raccoon is innocent, then the opossum is not lying.

If the rabbit is being truthful, then the opossum is lying.

If the rabbit is lying, then it has something to gain from lying.

The owl just found out that the rabbit would gain nothing from lying. Is the raccoon guilty or innocent? Why?

14) A car went around a one mile track at a speed of 30 miles per hour. At what speed must the car travel on its second time around the track if the driver wants to average 60 miles per hour for the two mile trip?

15) Twenty-three children are in a classroom. On the first day of school, the teacher asks what the chances are that two children in that class share the same birthday.

a) Very unlikely b) Small chance c) About a 50-50 chance d) Almost certain

### **Answers to 15 Perplexing Problems**

Warmups:

a) Throw the fan the opposite way you want to go. (Newtons Law: “For every action there is an equal and opposite reaction.”

b) 50 cents

Ball:  $n$

Glove:  $n + 9$

Equation:  $2n + 9 = 10$                        $n = \$.50$

c)  $10^{86}$

1)  $91/216$

You must find the chance of losing on each roll:  $5/6$

Chance of losing with three rolls:  $5/6 \times 5/6 \times 5/6 = 125/216$  If the chance of losing is  $125/216$ , then the chance of winning is  $1 - 125/216 = 91/216$

2) Keeps door:  $1/3$

Switches:  $2/3$

The correct answer is easier to see if you pretend that there are a thousand doors and the valuable prize is behind one of the doors. You are the contestant and pick door number 612. Your chances of winning are of

course 1 in 1000. Now the game show host, who knows where the prize is, opens the rest of the doors except door number 179. He now offers you the chance to change doors to number 179. Should you switch? You would be wise to switch because door number 179 has a 999 out of a thousand chance of winning, while your door still has only a 1 in 1000 chance of winning.

3) It is the same pressure either way.

3) Both are equally violent.

The severity of a crash depends on how quickly the person inside goes from 60 miles per hour to 0 miles per hour. In both situations, the deceleration is almost identical.

4) 1 in 3

Let's say that the boys and girls are coins. I ask you to throw two coins into the air and let them land on the table. I ask you one question. Is at least one of the coins heads? If you say yes, then I know the arrangement of coins on the table is either:

- 1) Heads      Tails
- 2) Tails      Heads
- 3) Heads      Heads

If at least one of the coins is heads, the chance of the other coin being heads is clearly 1 in 3. If one of the children is a boy, the chance of the other also being a boy is 1 in 3.

5) 4 mph

Most people will write 4.5 mph, but this is wrong. Because Phil spent more time traveling at 3 mph, this speed must be counted more. In this problem, he spent twice as much time going 3 mph as 6 mph so it would be counted twice as much. There are three blocks of time as shown below.

$$3 + 3 + 6 = 12 \qquad 12 \div 3 = 4$$

6) 2 in 3

This is a very counterintuitive problem. The best way to think of this is to ask yourself, before you pick a coin, what is the probability of picking a coin that has the same thing on both sides of the coin. (A heads or tails showing is equally likely so that doesn't change the original probability.) The answer of course is 2 in 3. Now you have a coin in your hand with heads showing. What is the possibility of it being a double sided coin. 2 in 3!!

$$\frac{t}{n} \text{ hours}$$

7)  $\frac{t}{n}$  method: Joseph can paint 2 cars in 10 hours. How long does it take him to paint one car?

The answer is clearly 5 hours: 10 hours  $\div$  2 cars = 5 hours.

Real problem:  $t$  hours  $\div$   $n$  cars =  $t/n$  hours

8) One hour 20 minutes

Think 1: Sara can paint  $1/2$  of the car in **one hour**. Dan can paint  $1/4$  of the car in **one hour**.

$1/2 + 1/4 = 3/4$  of the fence in the one hour. Each  $1/4$  is clearly 20 minutes. Remaining  $1/4$  would be 20 minutes.

9)  $47/80$

Find the chance of nobody snoring: Man 55% and woman 75%

One of each in a room---Chance of neither snoring:  $11/20 \times 3/4 = 33/80$

If the chance of neither is  $33/80$ , then the chance of someone snoring is

$$1 - 33/80 = 47/80$$

10) Choice 2: A raise of \$300 each six months

Look at the following chart that breaks the payments down into 6 month sections:

	Option of \$300 each 6 months	Option \$1000 each year
1st year: first 6 months	\$5,000	\$5,000
next 6 months	\$5,300 (\$300 raise)	\$5,000
2nd year: first 6 months	\$5,600	\$5,500
next 6 months	\$5,900	\$5,500
3rd year: first 6 months	\$6,200	\$6,000
next 6 months	\$6,500	\$6,000
4th year: first 6 months	\$6,800	\$6,500
next 6 months	\$7,100	\$6,500

As you can see, Mark would be much better off taking the \$300 raise every 6 months. The reason the brain has a hard time with this problem is that it jumps to the conclusion that a \$300 raise every 6 months is the same as a \$600 per year raise--it is not!

11) At first glance, it appears that you should switch. After all, there may be \$450 or \$1800 in the other box. It seems that you have an equal chance of losing \$450 or gaining an additional \$900. But that line of thinking would tell you to switch every time, which means that when you are picking Box A, you know you will end up with Box B. If you pick Box B, you know you will end up with Box A. Either way, you are just randomly picking a box.

12) Let's say Player B shoots one for 10 out of his first 10 shots. Now they are both shooting 10 for 40 or 25%. All the problem does is have Player B take more shots. The rest of his 20 shots he is 5 for 20, which maintains his 25% average.

13) Guilty

The only way the rabbit would be lying is if it had something to gain from lying.

The owl said that the rabbit had nothing to gain from lying, therefore the rabbit is telling the truth.

Because the rabbit is telling the truth, the opossum is lying.

If the raccoon was innocent, the opossum would not be lying.

Because the opossum is lying, the raccoon cannot be innocent, therefore it is guilty!!

14) It is impossible.

Using the formula: Distance = Speed x Time

The distance will be 2 miles and the speed is supposed to average 60 mph.

$$2 = 60 \times n \quad n = 1/30 \text{ of an hour or 2 minutes}$$

The amount of time will be 2 minutes, but the driver has already used that amount of time during his first trip around the track.

15) About a 50 -50 chance.

Remember, the question was not talking about a certain date. It was asking if any of the children would match with any of the others.

There are  $23 \times 22 = 506$  possible permutations of people in a room with 23 children: (1 with 2) (2 with 1) (1 with 3) (3 with 1) etc. We must divide the 506 by 2 because (1 with 2) and (2 with 1) are not really different when trying to match birthdays.  $506 \div 2 = 253$  combinations.

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